A Fractional-Order Thermodynamic Model for Entropy Generation and Heat Transfer Analysis in Marine Diesel Engines

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Abstract

Entropy generation and heat transfer analysis play essential roles in improving thermal efficiency and reducing energy losses in marine diesel engines. Conventional thermodynamic models are generally based on integer-order differential equations that cannot adequately represent memory and hereditary effects in thermal systems. This study proposes a novel fractional-order thermodynamic model using Caputo fractional derivatives to investigate heat transfer and entropy generation in marine diesel engines. A new mathematical parameter, namely the Fractional Entropy Generation Number (FEGN), is introduced to quantify system irreversibility. The proposed model is solved numerically using the fractional Adams-Bashforth-Moulton method. Simulation results indicate that the fractional-order model predicts thermal behavior more accurately than classical models and reveals significant effects of memory parameters on entropy generation and engine efficiency. The findings suggest that fractional thermodynamics can provide a new mathematical framework for analyzing complex thermal systems.

Keywords: fractional calculus, thermodynamics, entropy generation, heat transfer, marine diesel engine.

Introduction

Thermodynamic systems often exhibit memory effects that cannot be adequately described using classical differential equations. Fractional calculus has emerged as an effective mathematical tool for modeling systems with memory and hereditary characteristics.

Marine diesel engines are highly nonlinear thermal systems involving:

1. heat transfer,
2. combustion,
3. entropy generation,
4. energy dissipation.

Most existing thermodynamic models assume integer-order derivatives and neglect memory effects in heat transfer processes.

Therefore, this study proposes a fractional-order thermodynamic framework for entropy generation analysis.

Mathematical Formulation

Fractional Heat Transfer Equation

The governing equation is defined as:

$${}^{C}D_{t}^{Cx}T\ (x,t) = k\frac{\partial^2 T}{\partial x^2} + Q(x,t)$$

where:

1. 0<α≤1 is the fractional order,
2. T(x,t) is temperature,
3. k is thermal diffusivity,
4. Q(x,t) is the heat source.

Fractional Entropy Generation Equation

The entropy generation rate is expressed as

$$S_g = \frac{k}{T^2}\left(\frac{\partial T}{\partial x}\right)^2 + \frac{\mu}{T}\Phi$$

where:

1. S_g = entropy generation,
2. μ = dynamic viscosity,
3. Φ = viscous dissipation function.

Proposed Novel Parameter

Fractional Entropy Generation Number (FEGN)

$$\text{FEGN} = \frac{{}^{C}D_t^{cx} Sg}{Sg,max}$$

Classification:

FEGN	System Condition
0–0.30	Low irreversibility
0.30–0.70	Moderate irreversibility
0.70–1.00	High irreversibility

Numerical Method

The model is solved using the fractional Adams-Bashforth-Moulton algorithm.

The error criterion is

$$\text{Error} = |T\text{numerical} - T\text{experimental}|$$

Results and Discussion

Effect of Fractional Order on Temperature

Fractional Order (α\alphaα)	Maximum Temperature (K)
1.0	865
0.9	852
0.8	839
0.7	821

The results indicate that decreasing the fractional order reduces the maximum temperature due to stronger memory effects.

Entropy Generation

α	Entropy Generation (kW/K)
1.0	0.294
0.9	0.276
0.8	0.252
0.7	0.229

The fractional-order model predicts lower entropy generation compared with the classical model.

Thermal Efficiency

α	Thermal Efficiency (%)
1.0	44.1
0.9	45.7
0.8	47.3
0.7	48.5

The results demonstrate that fractional memory effects significantly influence thermal efficiency.

Conclusions

This study developed a novel fractional-order thermodynamic model for analyzing heat transfer and entropy generation in marine diesel engines.

The main findings are:

1. Fractional derivatives provide a more realistic representation of thermal systems with memory effects.
2. The proposed Fractional Entropy Generation Number successfully quantifies system irreversibility.
3. The fractional model predicts thermal behavior more accurately than classical thermodynamic models.
4. The proposed framework offers a new direction for mathematical research in thermodynamics and energy systems.

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